

Archdiocese of Chicago: Mathematics Curriculum Framework

Grade 12: Pre-Calculus

State Goal 6: Demonstrate and apply a knowledge and sense of numbers, including numeration and operations (addition, subtraction, multiplication, division), patterns, ratios, and proportions.

Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Critical for Mastery at Grade 12:</i></p> <p>12.06.01 Represent numbers in equivalent forms. (6A)</p> <p>12.06.02 Graph or interpret the graph of a complex number in rectangular and vector forms. (6A)</p> <p>12.06.03 Determine the opposite, reciprocal, absolute values, and positive integral powers of a complex number. (6A)</p> <p>12.06.04 Solve problems using complex numbers and their various representations. (6B)</p>	<p>Write the complex number $z = -2 - 2\sqrt{3}i$ in trigonometric form.</p> <p>Plot $i, i^2, i^3, i^4,$ and i^5 in the complex plane. Then write each in trig form. Describe what happens to the angle θ as higher powers of i are created.</p> <p>Given a complex number, the student should be able to analyze the number and state its opposite, reciprocal, absolute value, and positive integral powers.</p> <p>Using various representations of complex numbers, students should be able to solve a variety of problems. Students should be able to create their own problems and include an answer key. Exchange student's problems and have other students solve them. Students should also create a quiz for their classmates to take.</p>	<p>Connect to religion: Discuss the equivalent forms of Jesus as God and man.</p> <p>Research Benoit Mandelbrot and his Mandelbrot Set. Find an example of a fractal picture. Write a brief history of Mandelbrot and include an example of a fractal picture.</p>

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<p>12.06.05 Develop fluency in operations with real numbers, vectors, and matrices using mental computation or paper-and-pencil calculations for simple cases, and technology for more complicated cases. (6B)</p> <p>12.06.06 Describe the role of rounding error in calculations. (6C)</p>	<p>Students should calculate compounded daily interest $A = P \cdot e^{rt}$ where $e = 2.718$, for a variety of situations. In addition students should determine the rounding errors for these calculations. Afterwards, the students should determine the most accurate answer. (outcomes 05 and 06)</p>	

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Grade 12: Pre-Calculus Focus

State Goal 7: Estimate, make and use measurements of objects, quantities and relationships and determine acceptable levels of accuracy.

Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Critical for Mastery in Pre-Calculus:</i></p> <p>12.07.01 Set up and solve measurement conversions using multiple rates and conversion factors. (7A)</p> <p>12.07.02 Estimate the magnitude and directions of physical quantities (e.g., velocity, force, slope). (7B)</p> <p><i>Significant to Develop in Pre-Calculus:</i></p> <p>12.07.03 Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations. (7B)</p>	<p>A truck has a wheel measuring a 36” diameter. If the wheels are rotating at 630 rpm, find the truck’s speed in miles per hour.</p> <p>Viewing graphs of lines, state whether the slope is positive, negative, zero, or undefined. Mentally calculate the slope by counting the rise and run rather than drawing right triangles.</p> <p>If the area of a graphic region is bounded by the x-axis, the line $x=10$, and the graph of the equation $y=x^2$, approximate the area of the region using “bars” one unit wide.</p>	<p>Connect to science: Using a balloon that is partially inflated and placed near a heat source, record the temperature and the balloon’s size at various times (e.g., twenty minute intervals). Record your observations in U.S. customary and metric units.</p> <p>Connect to science: Investigate how Bernoulli’s Principle regarding velocity and properties of fluids applies to the ability of an airplane to rise into the air.</p> <p>Connect to business: Given the cost of a vehicle and the amount and number of monthly payments, use Newton’s method of successive approximations to find the monthly interest rate (accurate to five decimal places).</p>

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Grade 12 – Pre-Calculus

State Goal 8: Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems, and predict results.

Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Critical for Mastery at Grade 12:</i></p> <p>12.08.01 Interpret the overall relationship of two variables and connect it to one of the function families (linear, exponential, logarithmic, or power) from a graph. (8B)</p> <p>12.08.02 Relate the effect of transformations on graphs and equations. (8B)</p>	<p>Play a matching game in which students are given a group of different graphs (linear, exponential, logarithmic, or power) and they have to match the functions to the graphs (no calculator) by recognizing the properties that the graphs display of the relationship between two variables.</p> <p>Students describe what a graph of a function will look like using the equation of the function by applying the effect of different transformations onto the function.</p> <p>Students describe the transformations of a particular function by looking at the graph of the basic function and the complex version. For example, students would compare the graphs of $y = x^3$ and $y = -2(x - 5)^3 + 4$ and recognize the transformations that had occurred.</p>	<p>Have students find one career that is connected to any of the functions (linear, exponential, logarithmic, or power).</p>

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Learning Standard/Outcome	Sample Assessment	Connections
<p>12.08.03 Analyze functions by investigating domain, range, rates of change, intercepts, zeros, asymptotes, and local and global behavior. (8B)</p> <p>12.08.04 Describe and perform transformations, such as arithmetically combining, composing, and inverting commonly used functions using technology, to perform operations on more complicated symbolic operations. (8B)</p> <p>12.08.05 Describe and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions. (8C)</p>	<p>In groups, students are given a function, using their graphics calculator to assist them, the groups must analyze the function, identifying domain, range, rates of change, intercepts, zeros, asymptotes, and local and global behavior. Each group will then present their function and share their information.</p> <p>With a partner, students are given two functions. They must combine the two, compose the two, and invert the two. Using their graphics calculator they must draw pictures of the new graphs and analyze what they see.</p> <p>Divide the class into groups. Assign one of the classes of functions to each group. The groups must become experts on their type of function and they must teach the class all the properties of their function. (i.e., exponential, polynomial, rational, logarithmic, and periodic)</p>	<p>Relate to the importance of working together as a team. When in life beyond high school will this skill be important?</p>

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<p>12.08.06 Identify and explain the relationship between arithmetic/geometric sequences and linear/exponential functions. (8C)</p> <p>12.08.07 Apply sequences and their properties to solve real problems. (8C)</p> <p>12.08.08 Explain and apply relationships of x, y, and t in parametric equations. (8C)</p> <p>12.08.09 Solve problems using equations of exponential and logarithmic growth. (8D)</p>	<p>Using examples and discussions from previous class times, have students write a compare and contrast paper on the relationships between arithmetic/geometric sequences and linear/exponential functions. They must use examples and graphs.</p> <p>Solve application type problems using the properties of sequences.</p> <p>Using the internet, have students find an example of an application in which a parametric equation is necessary. Students must describe the situation and identify the relationships of x, y, and t in their example.</p> <p>Find real world applications of exponential growth and logarithmic growth and have students use the properties of these functions to solve the problems.</p>	<p>Connect the idea of a third variable to help describe two other unknowns to a science experiment.</p>

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<p>12.08.10 Describe the properties and features of any non-degenerate conic section from an equation or graph. (8B)</p> <p><i>Significant to Develop at Grade 12:</i></p> <p>12.08.11 Describe the limiting process using numerical analysis, graphs, and algebra. (8A)</p> <p>12.08.12 Write an equation for conic sections from a graph. (8B)</p>	<p>Set up stations around the classroom. Each station has either a non-degenerate conic section graph or equation. In groups of three have students walk around the classroom and identify the conic section and describe its properties and features.</p> <p>Given different functions in equation form, have students, with the aide of the graphics calculator, describe the limiting process on that particular function. Have students connect to visual (graph) to the analytical limit (numerical analysis and algebra).</p> <p>Use a graph of a conic section to determine the equation of the conic section.</p>	<p>Connect these different graphs and functions to art. Have students design a cover for an assembly or prayer service using pictures of conic sections.</p>

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Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Useful to Develop at Grade 12:</i></p> <p>12.08.13 Represent relationships arising from various contexts using symbolic expressions, including iterative and recursive forms. (8A)</p> <p>12.08.14 Translate between explicit and recursive forms of sequences where possible. (8A)</p>	<p>Give students different relationships from the content of Pre-Calculus (ex., functions, sequences, trigonometry, probability) and have them translate these in to symbolic (math) notation.</p> <p>Given an explicit form of a sequence have students translate it into a recursive form of a sequence.</p>	<p>Relate this to translating from Spanish to English. Why is this an important skill?</p> <p>Relate to the idea of cause and effect. How has this idea shaped our world?</p>

Archdiocese of Chicago: Mathematics Curriculum Framework
Grade 12: Pre-Calculus Focus

State Goal 9: Use geometric methods to analyze, categorize and draw conclusions about points, lines, planes and space.

Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Critical for Mastery at Grade 12:</i></p> <p>12.09.01 Analyze geometric situations using Cartesian coordinates and other coordinate systems such as navigational, polar, or spherical systems. (9A)</p> <p>12.09.02 Represent transformations of an object in the plane using function notation and matrices. (9A)</p> <p>12.09.03 Represent and describe with the language of geometry real-life objects, paths and regions in space. (9A)</p> <p>12.09.04 Extend the ideas of formal and informal proof to non-geometric situations. (9C)</p>	<p>Show that the graph of the equation $r=2a \sin\theta$ with $a>0$ is a circle of radius a and center at $(a,0)$, in rectangular coordinates.</p> <p>Describe the transformation formed by $y=cf(x-b)$</p> <p>Ellipses are the paths of planets and comets around the Sun. Describe the orbit of Earth if it has a semi-major axis of 149.598 Gm and an eccentricity of 0.0167.</p> <p>Reason through a solution in paragraph proof form.</p> <p>Using mathematical induction, prove: <i>3 is a factor of $n^3 + 2n$.</i></p>	<p>Explain how a global positioning system locates people in trouble.</p> <p>Connect to physics and the study of circuits.</p> <p>Visit Adler Planetarium. Give a report about the orbits using mathematical terms.</p> <p>Join the debate team. Read the Lincoln-Douglas Debates.</p>

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Grade 12: Pre-Calculus Focus

State Goal 9: Use geometric methods to analyze, categorize and draw conclusions about points, lines, planes and space.

Learning Standard/Outcome	Sample Assessment	Connections
<p>12.09.05 Describe the difference between an inductive argument and a deductive argument. (9C)</p>	<p>Explain the Principle of Mathematical Induction.</p>	
<p>12.09.06 Relate vector representation and trigonometric functions. (9D)</p>	<p>An airplane is flying on a bearing of 170 degrees at 460 mph. Find the component form of the velocity of the airplane.</p>	<p>Explain how vectors are used in the Physics course</p>
<p>12.09.07 Solve problems using vectors. (9D)</p>	<p>A force of 50 lb. acts on an object at an angle of 45 degrees. A second force of 75 lb. acts on the object at an angle of -30 degrees. Find the magnitude of the resultant force.</p>	<p>Find out how a sailor analyzes the effect of wind using vectors.</p>
<p>12.09.08 Relate circular functions, arcs, and radian measure to triangle trigonometry and degree measure. (9D)</p>	<p>Find $\sin \theta$ and $\tan \theta$ if $\cos \theta = 2/3$ and $\cot \theta$ is positive.</p>	
<p>12.09.09 Simplify expressions and solve problems using trigonometric identities. (9D)</p>	<p>Simplify: $\sqrt{1 - x^2}$, given $x = \cos \theta$</p>	

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<p>12.09.10 Solve trigonometric equations using circular functions. (9D)</p>	<p>Solve: $\cos^2 \theta + \cos \theta = \cos 2\theta$, for $[0, 2\pi]$</p>	<p>Research the National Oceanic and Atmospheric Administration and tides. Relate a trigonometric function with data that fits the tide behavior in Juneau, AL.</p>
<p>12.09.11 Identify key characteristics of graphs of trigonometric functions and their inverses. (9D)</p>	<p>Analyze the graph of $y = \sin x$, $y = \arcsin x$</p>	
<p>12.09.12 Graph trigonometric functions using translations and dilations. (9D)</p>	<p>Analyze the graph of $y = 5 \cos\left(3x - \frac{\pi}{6}\right) + 0.5$</p>	
<p>12.09.13 Graph a given trigonometric function using its characteristics (e.g., period, amplitude). (9D)</p>	<p>Construct and graph a sinusoid with amplitude of 3, period of π, and point on graph of $(0, 0)$.</p>	

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<p><i>Significant to Develop at Grade 12:</i></p> <p>12.09.14 Develop formal and informal proofs for three-dimensional figures. (9C)</p> <p>12.09.15 Rotate conic sections using trigonometric functions. (9D)</p>	<p>Using direction angles of vectors in space (α is the angle between \mathbf{v} and the positive x-axis, β is the angle between \mathbf{v} and the positive y-axis, and γ is the angle between \mathbf{v} and the positive z-axis, prove the Property of Direction Cosines: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$</p> <p>Express the equation $xy=1$ in terms of new $x'y'$ – coordinates by rotating the axes through an angle of 45 degrees.</p>	

Archdiocese of Chicago: Mathematics Curriculum Framework
Grade 12

State Goal 10: **Collect, organize and analyze data using statistical methods; predict results; and interpret uncertainty using concepts of probability.**

Learning Standard/Outcome	Sample Assessment	Connections
<p><i>Critical For Mastery at Grade</i></p> <p>12.10.01 Describe the differences among various kinds of studies and which types of inferences can legitimately be drawn from each other (10A)</p> <p>12.10.02 Describe how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.(10A)</p> <p>12.10.03 Determine the theoretical probability for a chance event using the binomial probability model. (10C)</p> <p>12.10.04 Identify patterns from a sample space. (10C) Describe the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.</p>	<p>Gather various data displays from newspapers , textbooks, and magazines and interpret the graphs and charts.</p> <p>From the data displays gathered determine which displays are inappropriate or misleading and how they could be improved.</p> <p>Given a data set such as the number of teams in the NCAA compute the appropriate probabilities such as: What is the probability that a team that has a name that starts with a vowel will win the championship game?</p>	<p>Gather statistics on the graduates of college in terms of salaries.</p> <p>Gather data related to studies involving tests on new drugs or medical procedures and discuss how ethical the studies are with regard to Catholic beliefs, traditions and morals.</p>

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<p>12.10.05 Present results and conclusions from given data using basic statistics (e.g., measures of central tendencies, standard deviation).</p> <p>12.10.06 Display a scatter plot, describe its shape and determine regression coefficients, regression equations and correlation coefficients for bivariate measurement data using technological tools.</p> <p>12.10.07 Formulate a question, design a study to answer the question, and collect data. (10B)</p> <p>12.10.08 Analyze potential methods of collecting information and decide which methods would produce the most reliable and accurate data. (10B)</p>	<p>Look at test data from a recent unit test from a science class or another discipline and display and analyze the data appropriately. Find data on grade point averages from several surrounding schools and make predictions about the school.</p> <p>Collect data on average family incomes from 1965 – 2000. (Or the number of shopping centers built in this time period, or the rise in cost of tuition, etc.). Compute appropriate statistics to determine if the gap between wealthy families and poor families increased, decreased or remained constant. Explore similar questions about other graphs that have been created using other data.</p> <p>Formulate a question regarding generating fractals. Collect information regarding the question, display the data, and analyze.</p> <p>Gather data the population in the school using a variety of methods. Evaluate the data and explain the reliability of data collection.</p>	<p>Gather data on missionaries in foreign countries and their effect on the populations that they serve.</p>

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<p>12.10.09 Analyze instruments used for surveys for errors and bias. (10B)</p> <p>12.10.10 Design and conduct experiments or simulations for probability, including the possible use of technology to simulate events. (10C)</p> <p><i>Significant to Develop at Grade</i></p> <p>12.10.11 Design a statistical experiment to answer a question about a realistic situation, conduct the experiment, use statistics to interpret the data, and communicate the results, individually and as members of a team. (10B)</p>	<p>Gather data on missionaries in foreign Countries and their effect on the Populations that they serve.</p> <p>Look at experiments performed in Science class or surveys from magazines and determine whether there are errors or bias in the collection, interpretation, or presentation of the data.</p> <p>Look at a set of test data from a class, determine what the data describes.</p>	

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<p>12.10.12 Explore the variability of a sample statistics from a known population and construct sampling distributions using simulations. (10B)</p> <p>12.10.13 Carry out a simulation to estimate probabilities, and if possible, compare it to the theoretical probability.</p> <p><i>Useful to Work On at Grade</i></p> <p>12.10.14 Interpolate, extrapolate, and make predictions from given information. (10A)</p> <p>12.10.15</p>	<p>Look at the grade point average of this year's basketball team. Look at the grade point average of the teams over the past 10 years. What conclusions can you draw?</p> <p>Devote class time to presentations given by students that show the results of a student designed and administered topic/survey. The survey should meet the expectations/requirements set forth in advance.</p> <p>Use the presentation described above to make predictions.</p>	

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<p>Evaluate survey results for conformity to simple distributions. (10A)</p> <p>12.10.16 Create a survey from a critical question and decide which sampling technique to use for the survey. (10B)</p> <p>12.10.17 Evaluate surveys for clarity, bias, return rate, and specialized audiences. (10B)</p>	<p>Review budget data from the US government to make predictions.</p> <p>Look at survey results obtained from a data set (from a magazine or off the web) and analyze what the data means.</p> <p>Develop a question appropriate for gathering data and design a survey such as a survey based on the value of catholic education or core requirement for graduation</p> <p>Administer the survey at a school function that includes parents, relatives, and students. Analyze the results applying the appropriate methods, display and communicate the results-display the information in a chart.</p>	